Emergence of classicality in quantum phase transitions

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We show that the long wavelength modes of a field become classical during a second order phase transition because of the interaction with the short wavelength modes of the field. In a massive scalar field model the number and thermal states of long wavelength modes, whose Wigner functions are sharply peaked around the classical trajectories during the phase transition, exhibit only classical correlation without achieving quantum decoherence. In a linearly coupled scalar field model, the long wavelength modes are shown to effectively achieve quantum decoherence because of the mode mixing. Finally we define a quantal ordering parameter that is linear in the field variable and satisfies the classical field equation.

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I. INTRODUCTION

A system becomes classical when it recovers classical correlation and loses quantum coherence, i.e., decoheres. The present Universe, for instance, is believed to have experienced a quantum-to-classical transition at a certain stage of its evolution from its early quantum state. One of such decoherence mechanisms is the interaction of the system with an environment [1–4]. The environment-induced decoherence mechanism explains correctly the density perturbation necessary for the structure formation of the present Universe [5]. A phase transition is another interesting phenomenon in which one observes the quantum-to-classical transition. For instance, the system of the long wavelength modes of a self-interacting scalar field become classical through the interaction with the environment of the short wavelength modes [6].

Another aspect of classicality is classical correlation. The parametric interaction of an open system provides classical correlation without any direct interaction with the environment. The coupling constants (parameters) of the open system change explicitly in time. A quantum field model in which the mass parameter changes sign during a quenched second order phase transition provides such an open system. In this model the dynamical evolution of phase transition is mainly described by a classical order parameter, whose quantum fluctuations are necessary for domains or topological defects [7]. In a previous paper [8] we introduced a quantum phase transition model without any classical order parameter in which a certain symmetric quantum state drives the phase transition. A question is then raised how the quantum phase transition exhibits classical features.

In the field model a classical background field is introduced as the order parameter, around which quantum field fluctuates [7]. The expectation value of the field, however, vanishes globally from the symmetry (parity) argument [6]. There is still room for the explanation of the order parameter in quantum theory: the condensation of the field into a coherent state having a nonzero expectation value. However, in Ref. [8] this initial quantum state is shown to be either a coherent or a coherent-thermal state. If the system starts initially from an exactly symmetric state such as the Gaussian vacuum or thermal equilibrium, the quantum law yields a zero expectation value throughout the phase transition. On the other hand, in Ref. [8] another possibility is discussed that the phase transition may proceed quantum mechanically in the symmetric quantum state, such as the Gaussian vacuum, number, and thermal states, even without globally forming a coherent condensate.

In this paper we shall show how classicality emerges from the symmetric quantum evolution of phase transition. Our model is a massive scalar field, the mass of which changes sign to mimick a quenched second order phase transition. This model initially has been used to study the behavior of an inflaton during the slow rollover in the new inflationary scenario [9] and also has been employed to describe quantum processes of the phase transition during the spinodal instability regime in Ref. [8]. It is known that the Gaussian state of the model obeys a classical probability distribution [9]. In this paper we shall apply the quantitative measure of both classical correlation and quantum decoherence in Ref. [10] to the quantum phase transition. For that purpose we derive the density matrices and Wigner functions for the Gaussian vacuum, number state, and thermal equilibrium.

It is found that the density matrix and Wigner function manifestly exhibit classical correlation for the long wavelength (soft) modes that grow exponentially, whereas both long and short wavelength (hard) modes maintain the same initial quantum coherence. Hence the massive scalar field does not achieve classicality in the genuine sense. To show

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how quantum decoherence occurs during the phase transition, we consider an analytically solvable model, motivated by a self-interacting scalar field, in which a long wavelength mode is linearly coupled to a short wavelength mode. It is shown that the mode mixing (coupling) between the long and short wavelength modes drastically reduces the degree of quantum coherence of the long wavelength mode during the phase transition. Thus the long wavelength mode becomes completely classical from the view point of both classical correlation and quantum decoherence. It is further suggested that a quantal quantity can be defined out of long wavelength modes to play a similar role of the order parameter.

The organization of this paper is as follows. In Sec. II, we derive the density matrices and the Wigner functions of a massive scalar field in the Gaussian vacuum, number state, and thermal equilibrium. In Sec. III, using the quantitative measure of classical correlation and quantum decoherence, we show that the unstable long wavelength modes exhibit classical correlation, whereas the short wavelength modes maintain their initial quantum coherence. In Sec. IV, we show how a long wavelength mode can achieve quantum decoherence during the phase transition through a linear coupling with a short wavelength mode. In Sec. V, we define a quantal order parameter that is linear in the field variable and satisfies the classical field equation.

II. DENSITY MATRIX AND WIGNER FUNCTION

As the first model for the second order phase transition, we consider a real massive scalar field with the Lagrangian density

$$\mathcal{L}=\frac{1}{2}[\dot{\phi}^2-(\nabla\phi)^2]-\frac{1}{2}m^2(t)\phi^2, \tag{1}$$

where the mass is assumed to change sign during the phase transition. Following Refs. [8] and [11], the action (1), upon a suitable mode decomposition, leads to the Hamiltonian

$$H(t)=\sum_{\alpha}\left[\frac{1}{2}\pi_{\alpha}^2(t)+\frac{1}{2}\omega_{\alpha}^2(t)\phi_{\alpha}^2(t)\right]=\sum_{\alpha}H_{\alpha}(t), \tag{2}$$

where

$$\omega_{\alpha}^2(t)=m^2(t)+\mathbf{k}^2, \tag{3}$$

and α denotes the Fourier mode defined by

$$\begin{aligned}\phi_{\mathbf{k}}^{(+)}(t)&=\frac{1}{2}[\phi_{\mathbf{k}}(t)+\phi_{-\mathbf{k}}(t)],\\ \phi_{\mathbf{k}}^{(-)}(t)&=\frac{i}{2}[\phi_{\mathbf{k}}(t)-\phi_{-\mathbf{k}}(t)].\end{aligned} \tag{4}$$

The Fourier modes (4) are Hermitian because $\phi_{\mathbf{k}}^{*}=\phi_{-\mathbf{k}}$. Thus the Hamiltonian (2) consists of the infinite number of decoupled, time-dependent oscillators. In quantum field approach the fundamental law is the functional Schrödinger equation (in units of $\hbar=k=1$)

$$i\frac{\partial}{\partial t}\Psi(\phi,t)=\hat{H}(t)\Psi(\phi,t). \tag{5}$$

As every mode is decoupled from each other, the wave functional to Eq. (5) is now given by the product

$$\Psi(\phi,t)=\prod_{\alpha}\Psi_{\alpha}(\phi_{\alpha},t) \tag{6}$$

of the wave functions of each Schrödinger equation

$$i\frac{\partial}{\partial t}\Psi_{\alpha}(\phi_{\alpha},t)=\hat{H}_{\alpha}(t)\Psi_{\alpha}(\phi_{\alpha},t). \tag{7}$$

The Fock space consisting of exact quantum states for a time-dependent oscillator was first constructed by Lewis and Riesenfeld [12] (for the references on many different methods, see [13]).

Now the quantum evolution of the Hamiltonian (2) is reduced to that of individual oscillators in Eq. (7). To find each Fock space, we follow the so-called Liouville–von Neumann (LvN) approach developed in Refs. [8] and [11,13]. In the LvN approach one finds a pair of operators for each mode of the Hamiltonian (2):

$$\begin{aligned}\hat{a}_{\alpha}(t)&=i[\varphi_{\alpha}^{*}(t)\hat{\pi}_{\alpha}-\dot{\varphi}_{\alpha}^{*}(t)\hat{\phi}_{\alpha}],\\ \hat{a}_{\alpha}^{\dagger}(t)&=-i[\varphi_{\alpha}(t)\hat{\pi}_{\alpha}-\dot{\varphi}_{\alpha}(t)\hat{\phi}_{\alpha}],\end{aligned} \tag{8}$$

that satisfy the quantum LvN equation

$$i\frac{\partial}{\partial t}\left\{\begin{matrix}\hat{a}(t)\\ \hat{a}^{\dagger}(t)\end{matrix}\right\}+\left[\left\{\begin{matrix}\hat{a}(t)\\ \hat{a}^{\dagger}(t)\end{matrix}\right\},\hat{H}_{\alpha}(t)\right]=0. \tag{9}$$

Equation (9) leads to the classical equation of motion for a complex φ_{α}:

$$\ddot{\varphi}_{\alpha}(t)+\omega_{\alpha}^2(t)\varphi_{\alpha}(t)=0. \tag{10}$$

By requiring the Wronskian condition

$$\dot{\varphi}_{\alpha}^{*}(t)\varphi_{\alpha}(t)-\dot{\varphi}_{\alpha}(t)\varphi_{\alpha}^{*}(t)=i, \tag{11}$$

one can make $\hat{a}_{\alpha}(t)$ and $\hat{a}_{\alpha}^{\dagger}(t)$ satisfy the standard commutation relation at any time

$$[\hat{a}_{\alpha}(t),\hat{a}_{\beta}^{\dagger}(t)]=\delta_{\alpha\beta}. \tag{12}$$

In the LvN approach the exact quantum state of the time-dependent oscillator is determined, up to some time-dependent factor, by the eigenstate of a Hermitian operator satisfying Eq. (9). For instance, the vacuum state is the zero-particle state $\hat{a}_{\alpha}(t)|0,t\rangle=0$ according to the standard quantum mechanics. Hence, as the phase transition proceeds, the time-dependent vacuum state at a late time is a squeezed state of the initial vacuum state [13,14].

As both operators in Eq. (8) already satisfy Eq. (9), we choose each number operator as

$$\hat{N}_\alpha(t) = \hat{a}^\dagger_\alpha(t)\hat{a}_\alpha(t), \tag{13}$$

and construct the Fock space consisting of the time-dependent number state

$$\hat{N}_\alpha(t)|n_\alpha,t\rangle = n_\alpha|n_\alpha,t\rangle. \tag{14}$$

Then the vacuum state is given by

$$\hat{a}_\alpha(t)|0_\alpha,t\rangle = 0, \tag{15}$$

and the number state by

$$|n_\alpha,t\rangle = \frac{1}{\sqrt{n_\alpha!}}[\hat{a}^\dagger_\alpha(t)]^{n_\alpha}|0_\alpha,t\rangle. \tag{16}$$

In Ref. [8] the number state has the coordinate representation

$$\Psi_{n_\alpha}(\phi_\alpha,t) = \left(\frac{1}{2\pi\varphi^*_\alpha\varphi_\alpha}\right)^{1/4}\frac{1}{\sqrt{2^{n_\alpha}n_\alpha!}}\left(\frac{\varphi_\alpha}{\varphi^*_\alpha}\right)^{n_\alpha/2} \times H_{n_\alpha}\left(\frac{\phi_\alpha}{\sqrt{2\varphi^*_\alpha\varphi_\alpha}}\right)\exp\left[\frac{i}{2}\frac{\dot{\varphi}^*_\alpha}{\varphi^*_\alpha}\phi^2_\alpha\right], \tag{17}$$

where H_{n_α} is a Hermite polynomial. From the definition $\rho_\Psi(x',x) = \langle x'|\Psi\rangle\langle\Psi|x\rangle$, we find the density matrix for the number state (17):

$$\rho_{n_\alpha}(\phi'_\alpha,\phi_\alpha,t) = \left(\frac{1}{2\pi\varphi^*_\alpha\varphi_\alpha}\right)^{1/2}\frac{1}{2^{n_\alpha}n_\alpha!} \times H_{n_\alpha}\left(\frac{\phi'_\alpha}{\sqrt{2\varphi^*_\alpha\varphi_\alpha}}\right)H_{n_\alpha}\left(\frac{\phi_\alpha}{\sqrt{2\varphi^*_\alpha\varphi_\alpha}}\right) \times \exp\left[-\frac{1}{2\varphi^*_\alpha\varphi_\alpha}\{\phi^2_{\alpha,C}+\phi^2_{\alpha,\Delta}\} + i\frac{d}{dt}\ln(\varphi^*_\alpha\varphi_\alpha)\phi_{\alpha,C}\phi_{\alpha,\Delta}\right], \tag{18}$$

where

$$\phi_{\alpha,C} = \frac{1}{2}(\phi'_\alpha+\phi_\alpha), \quad \phi_{\alpha,\Delta} = \frac{1}{2}(\phi'_\alpha-\phi_\alpha). \tag{19}$$

Once given the density matrix (18), one easily finds the Wigner function [15]

$$P_\alpha(\phi_\alpha,\pi_\alpha) = \frac{1}{\pi}\int_{-\infty}^{+\infty}dy\langle\phi_\alpha-y|\hat{\rho}_\alpha(t)|\phi_\alpha+y\rangle e^{2i\pi_\alpha y}. \tag{20}$$

Substituting Eq. (18) into Eq. (20), we obtain the Wigner function for $\hat{H}_\alpha$:

$$P_{n_\alpha}(\phi_\alpha,\pi_\alpha) = \frac{1}{\pi}(-1)^{n_\alpha}L_{n_\alpha}(8\varphi^*_\alpha\varphi_\alpha\tilde{H}_\alpha)e^{-4\varphi^*_\alpha\varphi_\alpha\tilde{H}_\alpha}, \tag{21}$$

where L_{n_α} is a Laguerre polynomial and

$$\tilde{H}_\alpha(\phi_\alpha,\pi_\alpha) = \frac{1}{2}\left[\pi_\alpha - \left\{\frac{d}{dt}\ln(\varphi^*_\alpha\varphi_\alpha)^{1/2}\right\}\phi_\alpha\right]^2 + \frac{1}{8(\varphi^*_\alpha\varphi_\alpha)^2}\phi^2_\alpha. \tag{22}$$

We now turn to the density matrix and Wigner function for the thermal state. In Refs. [8] and [11], the following density operator is used for each mode:

$$\hat{\rho}_{\mathrm{T}\,\alpha}(t) = 2\sinh\left(\frac{\beta\omega_{\alpha,i}}{2}\right)e^{-\beta\omega_{\alpha,i}(\hat{N}_\alpha(t)+1/2)}, \tag{23}$$

because $\hat{N}_\alpha(t)$ already satisfies the quantum LvN equation. Here the free parameters β and $\omega_{\alpha,i}$ are to be identified with the temperature and frequency, respectively, of the initial thermal equilibrium. From the density matrix of Ref. [1]

$$\rho_{\mathrm{T}\,\alpha}(\phi'_\alpha,\phi_\alpha,t) = \left[\frac{\tanh(\beta\omega_{\alpha,i}/2)}{2\pi\varphi^*_\alpha\varphi_\alpha}\right]^{1/2} \times \exp\left[i\frac{d}{dt}\ln(\varphi^*_\alpha\varphi_\alpha)\phi_{\alpha,C}\phi_{\alpha,\Delta}\right] \times \exp\left[-\frac{1}{2\varphi^*_\alpha\varphi_\alpha}\left\{\tanh\left(\frac{\beta\omega_{\alpha,i}}{2}\right)\phi^2_{\alpha,C} + \coth\left(\frac{\beta\omega_{\alpha,i}}{2}\right)\phi^2_{\alpha,\Delta}\right\}\right], \tag{24}$$

we obtain the Wigner function

$$P_{\mathrm{T}\,\alpha}(\phi_\alpha,\pi_\alpha) = \frac{1}{\pi}\tanh\left(\frac{\beta\omega_{\alpha,i}}{2}\right) \times \exp\left[-4\tanh\left(\frac{\beta\omega_{\alpha,i}}{2}\right)\varphi^*_\alpha\varphi_\alpha\tilde{H}_\alpha\right]. \tag{25}$$

Also, the density matrix (18) and Wigner function (21) for the Gaussian vacuum state ($n_\alpha = 0$) are obtained by taking the zero temperature limit $\beta\to\infty$ of Eqs. (24) and (25).

The nature of the number state (17) and thermal state (23) can also be understood from the Wigner functions (21) and (25). In particular, the 1-σ contour of the Wigner function contains some useful kinematical information of the quantum state in phase space. The 1-σ contour of the thermal state is defined by

$$\left\{\tanh\left(\frac{\beta\omega_{\alpha,i}}{2}\right)\right\}4\varphi^*_\alpha\varphi_\alpha\tilde{H}_\alpha = 1. \tag{26}$$

Here and hereafter the 1-σ contour of the Gaussian vacuum state is obtained by taking limit $\tanh(\beta\omega_{\alpha,i}/2)=1$. As Eq. (26) describes a rotated ellipse in the phase space (π_α,ϕ_α), we can find the major and minor axes through the transformation

$$\tilde{\pi}_\alpha=\cos\theta_\alpha\pi_\alpha-\sin\theta_\alpha\phi_\alpha,$$
$$\tilde{\phi}_\alpha=\sin\theta_\alpha\pi_\alpha+\cos\theta_\alpha\phi_\alpha, \tag{27}$$

where

$$\tan 2\theta_\alpha=\frac{2C_\alpha}{B_\alpha^+-B_\alpha^-},$$
$$B_\alpha^+=\left\{\tanh\left(\frac{\beta\omega_{\alpha,i}}{2}\right)\right\}2\varphi_\alpha^*\varphi_\alpha,$$
$$C_\alpha=\left\{\tanh\left(\frac{\beta\omega_{\alpha,i}}{2}\right)\right\}\varphi_\alpha^*\varphi_\alpha\frac{d}{dt}\ln(\varphi_\alpha^*\varphi_\alpha),$$
$$B_\alpha^-=\left\{\tanh\left(\frac{\beta\omega_{\alpha,i}}{2}\right)\right\}\left[\frac{1}{2\varphi_\alpha^*\varphi_\alpha}+\frac{1}{2}\varphi_\alpha^*\varphi_\alpha\left(\frac{d}{dt}\ln(\varphi_\alpha^*\varphi_\alpha)\right)^2\right]. \tag{28}$$

Then Eq. (26) is written in a canonical form

$$\left(\frac{\tilde{\pi}_\alpha}{\sqrt{1/\lambda_\alpha^+}}\right)^2+\left(\frac{\tilde{\phi}_\alpha}{\sqrt{1/\lambda_\alpha^-}}\right)^2=1, \tag{29}$$

where

$$\frac{1}{\lambda_\alpha^\pm}=\left(\frac{B_\alpha^++B_\alpha^-}{2}\right)\pm\left(\frac{B_\alpha^+-B_\alpha^-}{2}\right)\frac{1}{\cos\theta_\alpha}. \tag{30}$$

Physically, the constant area of the ellipse given by

$$A_\alpha=\frac{\pi}{\sqrt{\lambda_\alpha^+\lambda_\alpha^-}}=\coth\left(\frac{\beta\omega_{\alpha,i}}{2}\right), \tag{31}$$

implies that the phase transition only squeezes the initial quantum state of the massive scalar field unless there is a mode mixing (coupling). The entropy of each mode given by

$$S_\alpha\approx\ln A_\alpha=\ln\coth\left(\frac{\beta\omega_{\alpha,i}}{2}\right), \tag{32}$$

is also constant, i.e., isentropic, and vanishes for the vacuum state as expected. Without coarse graining the entropy of the massive scalar field does not change.

III. CLASSICAL CORRELATION OF LONG WAVELENGTH MODES

We now study classical correlation and quantum decoherence of the density matrix (18) or (24) and the Wigner function (21) or (25). The Wigner function (21) oscillates for an excited state and is not always positive definite, so it does not describe a true probability distribution in phase space. On the other hand, the Wigner function is positive definite for the Gaussian vacuum and thermal state. The thermal state is particularly of physical interest in the phase transition because it is mostly likely that the system starts from a thermal equilibrium.

A quantum system achieves quantum decoherence when the interference between classical trajectories is lost, and it recovers classical correlation when the Wigner function is peaked along a classical trajectory. The coherence length l_x of the density matrix (18) or (24) written in the form

$$\rho_{T_\alpha}(\phi',\phi_\alpha)=\left(\frac{\{\tanh(\beta\omega_{\alpha,i}/2)\}}{2\pi\varphi_\alpha^*\varphi_\alpha}\right)^{1/2}\exp[-\Gamma_{\alpha,C}\phi_{\alpha,C}^2-\Gamma_{\alpha,\Delta}\phi_{\alpha,\Delta}^2-\Gamma_{\alpha,M}\phi_{\alpha,C}\phi_{\alpha,\Delta}], \tag{33}$$

is roughly determined by the width of $\phi_{\Delta,\alpha}^2$:

$$l_x=\frac{1}{\sqrt{\Gamma_{\alpha,\Delta}}}=\left[\left\{\tanh\left(\frac{\beta\omega_{\alpha,i}}{2}\right)\right\}\varphi_\alpha^*\varphi_\alpha\right]^{1/2}. \tag{34}$$

Here the vacuum state result is $\tanh(\beta\omega_{\alpha,i}/2)=1$. Hence the coherence length increases or decreases depending on whether φ_α grows or decays. A large coherence length implies a large degree of quantum interference, so quantum decoherence is conditioned by the small magnitude of l_x.

More rigorously, the representation-independent measure of classical correlation of Ref. [10] is given by

$$\delta_{CC}=\sqrt{\frac{\Gamma_{\alpha,C}^2\Gamma_{\alpha,\Delta}^2}{\Gamma_{\alpha,M}^*\Gamma_{\alpha,M}}}=\frac{1}{4(\varphi_\alpha^*\varphi_\alpha)^2\left|\frac{d}{dt}\ln(\varphi_\alpha^*\varphi_\alpha)\right|}. \tag{35}$$

Similarly, the measure of quantum decoherence [10], which is determined by the ratio of the width of the off-diagonal element to that of the diagonal element, is given by

$$\delta_{QD}=\frac{1}{2}\sqrt{\frac{\Gamma_{\alpha,C}}{\Gamma_{\alpha,\Delta}}}=\frac{1}{2}\left\{\tanh\left(\frac{\beta\omega_{\alpha,i}}{2}\right)\right\}. \tag{36}$$

This means that the massive scalar field preserves the same initial quantum coherence throughout the phase transition.

The quantum system recovers classical correlation when $\delta_{CC}\ll 1$ and loses quantum coherence when $\delta_{QD}\ll 1$. So the system becomes classically correlated only when φ_α grows during the evolution. In the opposite case of decaying φ_α, the degree of classical correlation increases compared to that of quantum coherence, but the system retains quantum coherence. To see how the classical trajectory is recovered when $\delta_{CC}\ll 1$, we look into the Wigner function (25). Each contour of the Wigner function, which is quadratic π_α and

ϕ_α, depicts an ellipse in phase space. Suppose that both the classical field ϕ_α and auxiliary field φ_α grow exponentially during the phase transition:

$$\left\{\begin{matrix}\phi_\alpha(t)\\ \varphi_\alpha(t)\end{matrix}\right\}\simeq c_\alpha e^{f_\alpha(t)}, \tag{37}$$

then the classical trajectory obeys

$$\pi_\alpha(t)=\dot{\phi}_\alpha(t)\simeq \dot{f}_\alpha(t)\phi_\alpha(t). \tag{38}$$

On the other hand, the last term in Eq. (22) is exponentially suppressed compared to the first two terms and the overall coefficient of the exponent increases exponentially, so the Wigner function (25) is sharply peaked around a trajectory in phase space:

$$\pi_\alpha=\left[\frac{d}{dt}\ln(\varphi_\alpha^*\varphi_\alpha)^{1/2}\right]\phi_\alpha\simeq \dot{f}_\alpha(t)\phi_\alpha. \tag{39}$$

Indeed the Wigner function is sharply peaked around the classical trajectory.

Now we apply the criterion on classicality to the second order phase transition via an instantaneous quench, in which the mass changes sign as

$$m^2(t)=\begin{cases} m_i^2, & t<0,\\ -m_f^2, & t>0.\end{cases} \tag{40}$$

Before the quench, the most general solution to Eq. (10) is given by

$$\varphi_{\alpha,i}=\frac{1}{\sqrt{2\omega_{\alpha,i}}}[\cosh r_\alpha e^{-i\omega_{\alpha,i}t}+e^{-i\delta_\alpha}\sinh r_\alpha e^{i\omega_{\alpha,i}t}], \tag{41}$$

where

$$\omega_{\alpha,i}=\sqrt{m_i^2+\mathbf{k}^2}. \tag{42}$$

The system is stable and has the minimum expectation value when $r_\alpha=0$, corresponding to the true vacuum state. As $r_\alpha(\neq 0)$ represents a squeezing parameter of the true vacuum state, the vacuum state in Eq. (16) is the one-parameter squeezed vacua [13]. After the quench, the solution for each unstable long wavelength mode matches the initial solution (41) continuously at the onset of the quench $(t=0)$ and is given by

$$\varphi_{\alpha,U_f}=\frac{1}{\sqrt{2\omega_{\alpha,i}}}\Bigg[(\cosh r_\alpha+e^{-i\delta_\alpha}\sinh r_\alpha)\cosh(\tilde{\omega}_{\alpha,f}t) - i(\cosh r_\alpha-e^{-i\delta_\alpha}\sinh r_\alpha)\frac{\omega_{\alpha,i}}{\tilde{\omega}_{\alpha,f}}\sinh(\tilde{\omega}_{\alpha,f}t)\Bigg], \tag{43}$$

where

$$\tilde{\omega}_{\alpha,f}=\sqrt{m_f^2-\mathbf{k}^2}. \tag{44}$$

Whereas the solution for each stable short wavelength mode is given by

$$\varphi_{\alpha,S_f}=\frac{1}{\sqrt{2\omega_{\alpha,i}}}\Bigg[(\cosh r_\alpha+e^{-i\delta_\alpha}\sinh r_\alpha)\cos(\omega_{\alpha,f}t) - i(\cosh r_\alpha-e^{-i\delta_\alpha}\sinh r_\alpha)\frac{\omega_{\alpha,i}}{\omega_{\alpha,f}}\sin(\omega_{\alpha,f}t)\Bigg], \tag{45}$$

where

$$\omega_{\alpha,f}=\sqrt{\mathbf{k}^2-m_f^2}. \tag{46}$$

Thus, after the quench, the amplitude square of the long wavelength mode increases exponentially as

$$\varphi^*_{\alpha,U_f}\varphi_{\alpha,U_f}=\frac{1}{2\tilde{\omega}_{\alpha,f}}[\cosh(2r_\alpha)\cosh(2\tilde{\omega}_{\alpha,f}t) + \sin\delta_\alpha\sinh(2r_\alpha)\sinh(2\tilde{\omega}_{\alpha,f}t)]. \tag{47}$$

Therefore, the measure of classical correlation (35) for the long wavelength modes decreases exponentially to zero as the spinodal instability continues. Also, the Wigner function (21) is sharply peaked around the classical trajectory $\pi_\alpha \simeq\tilde{\omega}_{\alpha,f}\phi_\alpha$. On the contrary, the short wavelength modes have oscillating, bounded, amplitudes

$$\varphi^*_{\alpha,S_f}\varphi_{\alpha,S_f}=\frac{1}{2\omega_{\alpha,f}}[\cosh(2r_\alpha)\cos(2\omega_{\alpha,f}t) + \sin\delta_\alpha\sinh(2r_\alpha)\sin(2\omega_{\alpha,f}t)]. \tag{48}$$

Thus the measure of classical correlation (35) cannot become small for the short wavelength modes. But the measure of quantum decoherence (36) of the long and short wavelength modes remains constant.

In summary, the unstable long wavelength modes of the massive scalar field recover classical correlation during the phase transition whereas the stable short wavelength modes do not. However, both the long and short wavelength modes retain the same quantum coherence throughout the phase transition.

IV. QUANTUM DECOHERENCE DUE TO MODE MIXING

The long wavelength modes of the massive scalar field model in Sec. III gain classical correlation during the second order phase transition without achieving quantum decoherence. A more realistic model of the quantum phase transition is provided by Φ^4 theory with the Lagrangian density

$$\mathcal{L}=\frac{1}{2}[\dot{\phi}^2-(\nabla\phi)^2]-\frac{1}{2}m^2(t)\phi^2-\frac{\lambda}{4!}\phi^4, \tag{49}$$

where $m^2(t)$ is assumed to change sign as in Eq. (40). Upon decomposing the field into the Fourier mode (4), the Hamiltonian obtained from Eq. (49) takes the form [16]

$$H(t)=\sum_\alpha\left[\frac{1}{2}\pi_\alpha^2(t)+\frac{1}{2}\omega_\alpha^2(t)\phi_\alpha^2(t)\right]+\frac{\lambda}{4!}\left[\sum_\alpha \phi_\alpha^4(t)+3\sum_{\alpha\neq\beta}\phi_\alpha^2(t)\phi_\beta^2(t)\right], \quad (50)$$

where ω_α^2 is given by Eq. (3). The last coupled anharmonic terms prohibit any analytical solution. In the Gaussian approximation of the Hartree-Fock [7] or Liouville–von Neumann method [8], the number state of each mode is still given by Eq. (17), where φ_α now obeys the mean-field equation

$$\ddot{\varphi}_\alpha(t)+\left[\omega_\alpha^2(t)+\frac{\lambda}{2}\sum_\beta \varphi_\beta^*(t)\varphi_\beta(t)\right]\varphi_\alpha(t)=0. \quad (51)$$

In the Gaussian approximation the term ϕ^4 affects only the auxiliary variable φ_α through Eq. (51), but does not change the form of the state (17). Therefore, the Gaussian approximation for the self-interacting scalar field (49) does not lead to quantum decoherence. As the short wavelength modes provide an environment to the long wavelength modes, the mode mixing is expected to lead to quantum decoherence.

To analytically study the mode-mixing effect between a long and short wavelength mode, we consider an exactly solvable model

$$H_{\mathrm{M}}(t)=\frac{1}{2}\pi_S^2+\frac{1}{2}\omega_S^2(t)\phi_S^2+\frac{1}{2}\pi_H^2+\frac{1}{2}\omega_H^2(t)\phi_H^2+\lambda\phi_S\phi_H. \quad (52)$$

Here the subscript S and H denotes the long and short wavelength mode, respectively. As ω_S^2 and ω_H^2 depend on time through $m^2(t)$, the orthogonal transformation leading to normal modes depends time explicitly. Hence the product of the instantaneous eigenstates of each normal mode is not an exact quantum state of the original Hamiltonian (52). The Schrödinger equation has a Gaussian state of the form

$$\Psi_0(\phi_S,\phi_H,t)=N(t)\exp[-\{A_S(t)\phi_S^2+\lambda B(t)\phi_S\phi_H+A_H(t)\phi_H^2\}], \quad (53)$$

where N is the normalization constant, which is not of much concern to this paper. The coefficients determined by the Schrödinger equation are

$$A_S(t)=-i\frac{\dot{u}^*(t)}{2u^*(t)},$$

$$A_H(t)=-i\frac{\dot{v}^*(t)}{2v^*(t)},$$

$$B(t)=i\frac{\int u^*(t)v^*(t)}{u^*(t)v^*(t)}, \quad (54)$$

where

$$\ddot{u}(t)+\left[\omega_S^2(t)+\lambda^2\left\{\frac{\int u(t)v(t)}{u(t)v(t)}\right\}^2\right]u(t)=0, \quad (55)$$

$$\ddot{v}(t)+\left[\omega_H^2(t)+\lambda^2\left\{\frac{\int u(t)v(t)}{u(t)v(t)}\right\}^2\right]v(t)=0. \quad (56)$$

We solve Eqs. (55) and (56) separately for two cases: before and after the phase transition.

First, before the phase transition, the frequencies take the constant values $\omega_S^2(t)=\omega_{S,i}^2$ and $\omega_H^2(t)=\omega_{H,i}^2$. The solutions to Eqs. (55) and (56) are found to be

$$u(t)=\frac{1}{\sqrt{2\Omega_S}}e^{-i\Omega_S t},$$

$$v(t)=\frac{1}{\sqrt{2\Omega_H}}e^{-i\Omega_H t}, \quad (57)$$

where

$$\Omega_S^2=\omega_{S,i}^2-\left(\frac{\lambda}{\Omega_S+\Omega_H}\right)^2,$$

$$\Omega_H^2=\omega_{H,i}^2-\left(\frac{\lambda}{\Omega_S+\Omega_H}\right)^2. \quad (58)$$

Hence the Gaussian state is given by

$$\Psi_0(\phi_S,\phi_H)=N\exp\left[-\left\{\frac{\Omega_S}{2}\phi_S^2+\frac{\lambda}{\Omega_S+\Omega_H}\times\phi_S\phi_H+\frac{\Omega_H}{2}\phi_H^2\right\}\right], \quad (59)$$

and the reduced density matrix for the long wavelength mode by

$$\rho_{\mathrm{R}}(\phi_S',\phi_S)=\int_{-\infty}^{+\infty}d\phi_H\rho(\phi_S',\phi_H';\phi_S,\phi_H)=N^*N\sqrt{\frac{\pi}{\Omega_H}}\exp\left[-\left\{\Omega_S-\frac{\lambda^2}{\Omega_H(\Omega_S+\Omega_H)^2}\right\}\phi_{S,C}^2-\Omega_S\phi_{S,\Delta}^2\right]. \quad (60)$$

Finally, from the coefficients of the density matrix (33)

$$\Gamma_{S,C}=\Omega_S-\frac{\lambda^2}{\Omega_H(\Omega_S+\Omega_H)^2},\quad \Gamma_{S,\Delta}=\Omega_S,\quad \Gamma_{S,M}=0, \tag{61}$$

the measure of quantum decoherence is given by

$$\delta_{QD}=\frac{1}{2}\sqrt{\frac{\Gamma_{S,C}}{\Gamma_{S,\Delta}}}=\frac{1}{2}\sqrt{1-\frac{\lambda^2}{\Omega_S\Omega_H(\Omega_S+\Omega_H)^2}}, \tag{62}$$

and that of classical correlation by

$$\delta_{CC}=\sqrt{\frac{\Gamma_{S,C}^2\Gamma_{S,\Delta}^2}{\Gamma_{S,M}^*\Gamma_{S,M}}}. \tag{63}$$

Therefore, quantum decoherence is achieved by the mode mixing, but the characteristic behavior of classical correlation does not change.

Second, after the phase transition, Eqs. (55) and (56) become

$$\begin{aligned}&\ddot{u}(t)+\left[-\tilde{\omega}_{S,f}^2+\lambda^2\left\{\frac{\int u(t)v(t)}{u(t)v(t)}\right\}^2\right]u(t)=0\\ &\ddot{v}(t)+\left[\omega_{H,f}^2+\lambda^2\left\{\frac{\int u(t)v(t)}{u(t)v(t)}\right\}^2\right]v(t)=0.\end{aligned} \tag{64}$$

In the weak coupling limit $M\ll\tilde{\omega}_{S,f},\omega_{H,f}$, the approximate solutions are found to be

$$u(t)=c_1e^{\tilde{\Omega}_S t},\quad v(t)=c_2e^{-i\Omega_H t}, \tag{65}$$

where

$$\begin{aligned}\tilde{\Omega}_S=&\tilde{\omega}_{S,f}-\frac{\lambda^2}{2\tilde{\omega}_{S,f}}\left(\frac{\tilde{\omega}_{S,f}+i\omega_{H,f}}{\tilde{\omega}_{S,f}^2+\omega_{H,f}^2}\right)^2\\ &-i\frac{\lambda^4}{2\tilde{\omega}_{S,f}^2\omega_{H,f}}\left(\frac{\tilde{\omega}_{S,f}+i\omega_{H,f}}{\tilde{\omega}_{S,f}^2+\omega_{H,f}^2}\right)^4\\ &-\frac{\lambda^4}{8\tilde{\omega}_{S,f}^3}\left(\frac{\tilde{\omega}_{S,f}+i\omega_{H,f}}{\tilde{\omega}_{S,f}^2+\omega_{H,f}^2}\right)^4,\\ \tilde{\Omega}_H=&\omega_{H,f}+\frac{\lambda^2}{2\omega_{H,f}}\left(\frac{\tilde{\omega}_{S,f}+i\omega_{H,f}}{\tilde{\omega}_{S,f}^2+\omega_{H,f}^2}\right)^2\\ &+i\frac{\lambda^4}{2\tilde{\omega}_{S,f}\omega_{H,f}^2}\left(\frac{\tilde{\omega}_{S,f}+i\omega_{H,f}}{\tilde{\omega}_{S,f}^2+\omega_{H,f}^2}\right)^4\\ &-\frac{\lambda^4}{8\omega_{H,f}^3}\left(\frac{\tilde{\omega}_{S,f}+i\omega_{H,f}}{\tilde{\omega}_{S,f}^2+\omega_{H,f}^2}\right)^4.\end{aligned} \tag{66}$$

These solutions are accurate at late times provided that $\mathrm{Re}(\tilde{\Omega}_S)t\gg1$. Then, the reduced density matrix is given by

$$\begin{aligned}\rho_{\mathrm{R}}(\phi_s',\phi_S)=&N^*N\sqrt{\frac{\pi}{A_H^*+A_H}}\exp[-\Gamma_{S,C}\phi_{S,C}^2-\Gamma_{S,\Delta}\phi_{S,\Delta}^2\\ &-\Gamma_{S,M}\phi_{S,C}\phi_{S,\Delta}],\end{aligned} \tag{67}$$

where

$$\Gamma_{S,C}=2\left[\mathrm{Re}\,A_S-\frac{\lambda^2(\mathrm{Re}\,B)^2}{4\,\mathrm{Re}\,A_H}\right], \tag{68}$$

$$\Gamma_{S,\Delta}=2\left[\mathrm{Re}\,A_S+\frac{\lambda^2(\mathrm{Im}\,B)^2}{4\,\mathrm{Re}\,A_H}\right], \tag{69}$$

$$\Gamma_{S,M}=4i\left[\mathrm{Im}\,A_S-\frac{\lambda^2(\mathrm{Re}\ \mathrm{B})(\mathrm{Im}\,B)}{2\,\mathrm{Re}\,A_H}\right]. \tag{70}$$

To order λ^4, the coefficients are approximated by

$$\Gamma_{S,C}=\frac{5\lambda^4\omega_{H,f}(\tilde{\omega}_{S,f}-\omega_{H,f})}{2\tilde{\omega}_{S,f}(\tilde{\omega}_{S,f}^2+\omega_{H,f}^2)^4}, \tag{71}$$

$$\begin{aligned}\Gamma_{S,\Delta}=&\frac{\lambda^2}{\omega_{H,f}(\tilde{\omega}_{S,f}^2+\omega_{H,f}^2)}\left[1+\frac{\lambda^2\omega_{H,f}}{(\tilde{\omega}_{S,f}^2+\omega_{H,f}^2)^3}\right.\\ &\left.\times\left(\frac{\tilde{\omega}_{S,f}^4}{2\omega_{H,f}^3}-\frac{3\tilde{\omega}_{S,f}^2}{\omega_{H,f}}-\frac{5\omega_{H,f}^2}{2\tilde{\omega}_{S,f}}+\omega_{H,f}\right)\right],\end{aligned} \tag{72}$$

$$\begin{aligned}\Gamma_{S,M}=&-2i\tilde{\omega}_{S,f}\left[1+\frac{\lambda^2\omega_{H,f}^2}{2\tilde{\omega}_{S,f}^2(\tilde{\omega}_{S,f}^2+\omega_{H,f}^2)^2}\right.\\ &\left.+\frac{\lambda^4}{(\tilde{\omega}_{S,f}^2+\omega_{H,f}^2)^4}\left(\frac{16}{5}-\frac{3\omega_{H,f}^2}{2\tilde{\omega}_{S,f}^2}-\frac{\omega_{H,f}^4}{4\tilde{\omega}_{S,f}^4}\right)\right].\end{aligned} \tag{73}$$

Therefore, to λ^4 order, the measure of quantum decoherence is given by

$$\delta_{QD}=\frac{\lambda}{2}\sqrt{\frac{5\omega_{H,f}^2(\tilde{\omega}_{S,f}-\omega_{H,f})}{2\tilde{\omega}_{S,f}(\tilde{\omega}_{S,f}^2+\omega_{H,f}^2)^2}}, \tag{74}$$

and that of classical correlation by

$$\delta_{CC}=\frac{5\lambda^6}{4}\,\frac{|\tilde{\omega}_{S,f}-\omega_{H,f}|}{\tilde{\omega}_{S,f}^2(\tilde{\omega}_{S,f}^2+\omega_{H,f}^2)^5}. \tag{75}$$

A few comments are in order. First, quantum decoherence is observed even for the quadratic Hamiltonian (52). It is a consequence of the mode-mixing and does not exclusively pertain to the nonlinearity of the system. For the quadratic system with constant parameters, the parameters introduced in Ref. [4] as

$$e^{\eta}=\sqrt{\frac{\lambda_+}{\lambda_-}},\quad \tan(2\theta)=\frac{\lambda}{\omega_H^2-\omega_S^2}, \tag{76}$$

where

$$\lambda_{\pm}=\frac{1}{2}[\omega_S^2+\omega_H^2\pm\sqrt{(\omega_S^2-\omega_H^2)^2+\lambda^2}], \tag{77}$$

yield another expression for quantum decoherence

$$\delta_{QD}=\frac{1}{2}\sqrt{\frac{1}{\cosh^2\eta-\sinh^2\eta\cos^2(2\theta)}}. \tag{78}$$

Obviously, there is no quantum decoherence, $\delta_{QD}=1/2$, for the zero mode-mixing $\lambda=\theta=0$. In the limiting case of weak coupling $\lambda\ll|\omega_H^2-\omega_S^2|$, the mixing angle is small ($\theta\approx 0$) and the measure of quantum decoherence becomes $\delta_{QD}\approx 1/2$, which is consistent with the result (62). Whereas, for the two identical oscillators $\omega_S=\omega_H$ with the maximum mixing angle $\alpha=\pi/4$, the measure is simply given by

$$\delta_{QD}=\frac{1}{2\cosh\eta}, \tag{79}$$

where

$$e^{\eta}=\sqrt{\frac{2\omega_S^2+\lambda}{2\omega_S^2-\lambda}}. \tag{80}$$

Second, it is shown that the unstable system efficiently achieves quantum decoherence through the coupling with an environment. This effect is also observed in the Bateman or the Feshbach-Tikochinsky oscillator. The Bateman or Feshbach-Tikochinsky oscillator is a conserved two-oscillator system which consists of a damped and an amplified oscillator [17,18]. The amplified oscillator has an exponentially increasing amplitude at the expense of the damped oscillator and reminds us of the unstable mode of the model (52). It is found that the amplified oscillator shows a similar quantum decoherence [19]. Thus it may be inferred that even in quadratic systems quantum decoherence is a consequence of the mode mixing (coupling) of systems to the environment.

V. QUANTAL ORDER PARAMETER

In the previous section it is found that the unstable long wavelength modes recover their classical correlation but the stable short wavelength modes retain the quantum coherence during the quench. Thus the parametric interaction via the quench makes the quantum phase transition exhibit classicality. As some, though not all, of the essential features of the second order phase transition can also be explained by classical theory, it would be interesting to find a quantal quantity that behaves like a classical order parameter.

In quantum theory the order parameter is frequently defined by $\langle\hat{\phi}(t,\mathbf{x})\rangle$, the mean value of the field. The mean value of a coherent state satisfies the classical field equation with quantum correction from fluctuations. However, the thermal state or Gaussian vacuum studied in this paper is symmetric, so the expectation value of the field modes (field) with respect to this state vanishes:

$$\langle\hat{\phi}_\alpha\rangle_{\mathrm{T}}=\mathrm{Tr}(\hat{\phi}_\alpha\hat{\rho}_{\mathrm{T},\alpha})=0. \tag{81}$$

The nonzero expectation values come from the quadratic moment of the field or field modes. One candidate is the two-point function $\langle\hat{\phi}(t,\mathbf{x})\hat{\phi}(t,\mathbf{y})\rangle$. It satisfies the classical field equation in the case of a massive scalar field but not in the case of a self-interacting scalar field. As the two-point function is quadratic in the field variable, we wish to define a quantity that is linear in the field variable and satisfies the classical field equation.

We note that the quadratic moment of the field mode also has the nonzero expectation value

$$\langle\hat{\phi}_\alpha^2\rangle_{\mathrm{T}}=\varphi_\alpha^*\varphi_\alpha\coth\left(\frac{\beta\omega_{\alpha,i}}{2}\right). \tag{82}$$

The classical equation (10) for a complex φ_α leads to the equation [8]

$$\ddot{\xi}_\alpha(t)+\omega_\alpha^2(t)\xi_\alpha(t)-\frac{1}{4\xi_\alpha^3(t)}=0, \tag{83}$$

where ξ_α is the amplitude of complex φ_α:

$$\varphi_\alpha(t)=\xi_\alpha(t)e^{i\theta_\alpha(t)}. \tag{84}$$

The last term in Eq. (83), which originates from quantization rule and guarantees the Heisenberg uncertainty principle, can be neglected for the long wavelength modes because ξ_α increases exponentially during the quench. Therefore, it approximately satisfies

$$\ddot{\xi}_\alpha(t)+\omega_\alpha^2(t)\xi_\alpha(t)\simeq 0. \tag{85}$$

As each $\xi_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}}$ satisfies the classical field equation, it follows that the quantal quantity defined by

$$\phi_{\mathrm{O}}(t,\mathbf{x})=\int_{k=0}^{k\leqslant m_f}\frac{d^3\mathbf{k}}{(2\pi)^3}\xi_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}} =\int_{k=0}^{k\leqslant m_f}\frac{d^3\mathbf{k}}{(2\pi)^3}\left[\langle\hat{\phi}_\alpha^2\rangle_{\mathrm{T}}\tanh\left(\frac{\beta\omega_{\alpha,i}}{2}\right)\right]^{1/2}e^{i\mathbf{k}\cdot\mathbf{x}} \tag{86}$$

also satisfies the classical field equation

$$\ddot{\phi}_{\mathrm{O}}(t,\mathbf{x})-\nabla^2\phi_{\mathrm{O}}(t,\mathbf{x})+m^2(t)\phi_{\mathrm{O}}(t,\mathbf{x})\simeq 0. \tag{87}$$

It should be remarked that ϕ_{O} is linear in the field variable and is defined out of the long wavelength modes, which become classically correlated during the quench. In this sense ϕ_{O} carries many features of the classical order parameter.

VI. CONCLUSION

We studied classical correlation and quantum decoherence in quantum phase transition. For the initial state such as the Gaussian vacuum or number state or thermal equilibrium, the expectation value of the field vanishes throughout the phase transition. Hence it cannot be used as the order parameter. The phase transition in such a symmetric state, however, can be explained entirely within the framework of quantum field theory. In the massive scalar field model the long wavelength modes become unstable and grow exponentially during the quench. The quantitative measure from the density matrix shows classical correlation of the long wavelength modes during the phase transition. Likewise, the Wigner functions for these modes are sharply peaked around their classical trajectories and thus confirm classical correlation. Whereas, the short wavelength modes are stable throughout the quench process and retain the quantum coherence.

Classicality is, in a strict sense, conditioned not only by the recovery of classical correlation but also by the loss of quantum coherence. However the massive scalar model does not achieve completely quantum decoherence, because the measure of quantum decoherence keeps the value determined by the initial Gaussian vacuum or thermal equilibrium. Hence the system recovers classicality partially, depending on the relative magnitude between the measures of classical correlation and quantum decoherence. To achieve quantum decoherence it is necessary for the long wavelength modes to couple to an environment. The environment-induced decoherence was studied numerically in the self-interacting scalar field model, where the long wavelength modes are coupled to the short wavelength modes [6,20]. However, it is very hard to find any analytical solution of the self-interacting scalar model. The Gaussian approximation of the Hartree-Fock [7] or the Liouville–von Neumann method [8] does not include the mode mixing effect appropriately to explain quantum decoherence.

In this paper, to analytically study the mode mixing effect, we turned on a linear coupling between the long and short wavelength modes of the massive scalar field. This exactly solvable model shows that the long wavelength modes indeed decohere because of the mode-mixing with the short wavelength modes. Therefore, classicality emerges during the phase transition from the unstable long wavelength modes coupled to the stable short wavelength modes. These long wavelength modes behave classically and provide a quantal quantity that is linear in the field variable, satisfies the classical field equation, and thus behaves like a classical order parameter.

After this paper was completed, we were informed of Ref. [21], which also shows quantum decoherence of the long wavelength modes through the nonlinear coupling to the short wavelength modes in the self-interacting scalar field and confirms the numerical result [6]. However, any analytical solution in a closed form has not been found for the self-interacting scalar field. At best, the analytical solution can be found by perturbatively including the mode-mixing terms [16], which is beyond the scope of this paper. The non-Gaussian effect from the mode-mixing is expected to result in quantum decoherence. As quantum decoherence is largely a consequence of mode-mixing (coupling), our exactly solvable model may shed some light in understanding the mechanism of classicality.

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